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Dynamic Stability of Shallow Shells Subjected to Follower Forces

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Dynamic stability of cylindrically curved shallow shells subjected to follower forces is presented. The shell analysis uses a two-field variable variational principle with the transverse displacement and Airy stress function as field variables. Starting from the variational equation of the problem, the Euler-Lagrange equations governing the problem and the boundary conditions on the transverse displacement and the Airy stress function are obtained. A finite element formulation that preserves C^1 continuity is used for the solution of the problem. Numerical results are given and the results obtained are discussed. It is shown that the use of the two-field variable variational principle permits, through static condensation, a reduction in the effort needed for the solution of the stability problem to that of a flat plate. Similarity between the response of a flexible structure to a follower force and the aerodynamic supersonic panel flutter is discussed. Practical applications of the problem are large space structures, such as solar power stations, which can undergo flutter or divergence instabilities due to their low rigidity when thrusted by follower forces.

Nomenclature

A	= shell total surface area
a, b	= shell dimensions in the x and y directions, respectively
C_1, C_2	= constants proportional to the external follower forces
D	= shell flexural rigidity, $Eh^3/12(1 - \nu^2)$
E	= Young's modulus
F	= Airy stress function
H_{mn}	= first order Hermitian polynomials
h	= shell thickness
$[K]$	= system stiffness matrix
$[k]$	= element stiffness matrix
M_x, M_y, M_{xy}	= bending stress resultants
$[M]$	= system mass matrix
$[m]$	= element mass matrix
$[N_1]$	= system incremental stiffness matrix
$[n_1]$	= element incremental stiffness matrix
$[N_2]$	= system damping matrix
$[n_2]$	= element damping matrix
$P(w)$	= follower force acting on the boundary s
$Q_t(w)$	= follower force distributed on the shell surface, function of time
$Q_x(w)$	= follower force distributed on the shell surface, function of space coordinate
R	= cylindrically curved shell radius
t	= time
u, v	= shell inplane middle surface displacements in the x and y directions
w	= shell transverse displacement
x, y	= shell middle surface coordinates in the axial and the circumferential directions
z	= shell curvature parameter, $(1 - \nu^2)a^2/Rh$
δ	= variational operator
ν	= Poisson's ratio
Π^*	= Reissner functional
ρ	= material mass density per unit area
∇^2	= $(\partial^2/\partial x^2 + \partial^2/\partial y^2)$
$()_{,x}$	= $\partial()/\partial x$
$(\dot{\ })$	= partial derivative with respect to time

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I. Introduction

SINCE Beck[1] published his classical paper on elastic stability of beams subjected to follower forces in 1952, many authors investigated the problem of stability of structures under the effect of nonconservative loads. Ziegler[2] introduced the concept of circulatory loads by assuming that the nonconservative forces are only proportional to the generalized coordinates and do not depend on their derivatives with respect to time. Hermann[3] discussed and classified the nonconservative loads acting on structures according to their origin. A basic characteristic feature of the behavior of structures under the action of nonconservative follower loads is that the structures may fail due to dynamic or static instabilities. These two modes of failure have been termed in the literature as flutter or divergence modes of failure, respectively, by analogy to the instabilities occurring in aeroelastic analyses. The first applications in the stability of structures subjected to nonconservative follower forces treated one-dimensional structures, namely, cantilever beams subjected to end follower forces.[4–7] Free-free beams subjected to end follower thrust without[8–10] and with[11,12] control for stability augmentation were further studied. Free-free beams subjected to follower forces on both ends were investigated by Celep in Refs. 13 and 14. Starting from the 1970s, several research studies on the stability of two-dimensional structures, namely, flat plates, subjected to follower forces were performed. Petterson[15] and Farshad[16] studied the stability of flat plates under the action of subtangential follower forces. Leipholz[17,18] and Leipholz and Waddington[19] studied the problem of simply supported rectangular plates under the action of a distributed tangential follower load, and rectangular plates simply supported on three sides and free on the fourth side where the tangential follower force acts. Static divergence instabilities were reported to occur first for the geometry and the cases studied in his investigations.[17–19] Culkowski and Reismann[20] reported dynamic instabilities for clamped-free flat rectangular plates with two opposite simply supported edges and the follower force applied on the free edge. Adali[21] investigated the stability regions of rectangular flat plates subjected to follower forces and unidirectional axial forces. Results for clamped-free plates with the two other edges simply supported were reported. Higushi and Dowell[22] investigated the dynamic stability of rectangular flat plates that have all four edges free and are subjected to follower forces on one edge. In Ref. 23, Higushi and Dowell included in the analysis the effect of damping in the equation of motion. Their analyses were performed in a free vibration modal base. Review papers covering the period to 1967 and to 1975 on the stability of structures subjected to follower forces are given in Refs. 24 and 25, respectively. Two recent books by Huseyin[26] and Leipholz[27] provide an up-to-date treatment of the elastic stability of nonconservative systems.

Structural analysis of plates and shells using two-field variables, the transverse displacement w and Airy stress function F, has been investigated by several authors. Donnell,[28] Marguerre,[29] and von Kármán and Tsein[30] made the major contributions in obtaining the differential equations for static analysis. Reissner[31] introduced a variational principle with w and F being the field variables. Based on an order of magnitude analysis to justify the omission of the in-plane inertia terms, Reissner[32] extended the application to the dynamic analysis of shallow shells. In spite of the simplification it introduces, Reissner's two-field variable principle is scarcely used in finite element applications. The main reason is attributed to the complications introduced for the application of the boundary conditions on the Airy stress function.[33,34] In Ref. 35, starting from Reissner's variational equation for free vibration of thin cylindrical curved plates, the Euler-Lagrange equations governing the problem and the boundary conditions were obtained. It was shown that the boundary conditions on F are as simple and direct to apply as on w. The variational principle was used to derive a C^1 continuity rectangular finite element, and numerical results of free vibration for freely supported square curved plates were presented. In Ref. 36, the formulation was extended to the buckling analysis of cylindrically curved panels. Numerical results were presented for various classical end conditions and for different aspect ratios of the panels. In Ref. 37, the formulation was applied to the supersonic flutter of cylindrically curved panels. Numerical results were presented for square panels with flow in the meridional direction for clamped, simply supported, and freely supported end conditions. In Ref. 38, the formulation was generalized, and the results of Refs. 35–37 were amplified. Several numerical solutions were presented and were compared with previous analytical solutions, numerical calculations, and experimental findings.

The purpose of the present work is to study the stability of cantilever cylindrically curved panels subjected to nonconservative tangential follower forces distributed over the area of the panel and a distributed follower at the free end of the panel. The analysis presented is based on Reissner's two-field variable variational principle. The solution of the problem is made using a C^1 continuity finite element method. It is shown that the computational effort, when the present solution is used, is equivalent to the effort required for a flat plate solution. The instabilities are obtained for several boundary conditions of the lateral edges of the cantilever cylindrically curved panel. Namely, results are obtained for the lateral edges free, simply supported, freely supported, and clamped. The effect of the panel aspect ratio and the shell curvature are studied for all of the boundary conditions already mentioned.

The similarity between the response of the shell to nonconservative follower forces and its response in the presence of a supersonic flow is presented and discussed. The dynamic instability here treated is due to the presence of nonconservative follower forces. In aeronautical engineering the source of nonconservative forces is often an aerodynamic flow. The literature of aerodynamic flutter and divergence is extensive. Specifically, for the case of aeroelasticity of plates and shells, many parameters affecting the instability mechanism, e.g., flexibility of the supports, use of modern materials, structural nonlinearity, flow orientation, etc. have been amply studied. This is not the case of instability due to follower forces, where the literature lacks this deep study. In the present investigation the similarity between the two problems is demonstrated and, thus, all of the experience already accumulated in the field of aeroelasticity of plates and shells can be directly applied to the problem of the stability of the shell in the presence of the nonconservative follower forces and, therefore, duplication of the effort is avoided.

Practical applications of flexible structures subjected to follower forces are ample. Among these applications one can cite the uniformly accelerated slender flexible missiles subjected to follower thrust acting on the free end, directional controlled elastic rockets idealized as free-free flexible beams subjected to follower forces. Other examples are plate-like space structures, constructed with large number of repeated frames, which can be modeled as panel structures with different boundary conditions applied at the edges and subjected to follower forces.Thrusted exhaust panels are examples of cantilever cylindrically curved shells having different boundary conditions on the lateral edges and are subjected to nonconservative follower forces.

II. Problem Formulation

The variational equation of thin cylindrically curved shallow shells,[31] neglecting the effect of in-plane inertias[32] and considering the effect of the work done by external nonconservative follower forces can be expressed as

$$\begin{aligned}\delta(\Pi^*) = \delta \int_t \Bigg\{ \frac{1}{2}\int_A \rho h \dot{w}^2\,\mathrm{d}A - \frac{D}{2}\int_A \big[w_{,xx}^2 + w_{,yy}^2 \\ + 2\nu w_{,xx}w_{,yy} + 2(1-\nu)w_{,xy}^2\big]\mathrm{d}A + \frac{1}{2Eh}\int_A \big[F_{,xx}^2 + F_{,yy}^2 \\ - 2\nu F_{,xx}F_{,yy} + 2(1+\nu)F_{,xy}^2\big]\mathrm{d}A - \int_A \frac{w}{R}F_{,xx}\,\mathrm{d}A\Bigg\}\,\mathrm{d}t \\ + \int_t \Bigg\{\int_A [Q_t(w) + Q_x(w)]\delta w\,\mathrm{d}A + \int_s P(w)\delta w\,\mathrm{d}s\Bigg\}\,\mathrm{d}t \\ = 0 \qquad (1)\end{aligned}$$

where the functions subjected to variation are the transverse displacement w and the Airy stress function F. Performing the variational operation, grouping terms, and applying Green's theorem, the variational equation governing the problem reads

$$\begin{aligned} -\int_A \delta w\Big[D\nabla^4 w + \frac{1}{R}F_{,xx} + \rho h w_{,tt} + Q_t(w) \\ + Q_x(w) + P(w)\Big]\mathrm{d}A + \int_A \delta F\Big[\frac{1}{Eh}\nabla^4 F - \frac{1}{R}w_{,xx}\Big]\mathrm{d}A \\ - \int_C \delta w\big[M_{x,x} + 2M_{xy,y}\big]\,\mathrm{d}y + \int_C \delta w\big[M_{y,y} + 2M_{xy,x}\big]\,\mathrm{d}x \\ + \int_C \delta w_{,x}M_x\,\mathrm{d}y - \int_C \delta w_{,y}M_y\,\mathrm{d}x + \int_C [M_{xy}\delta w]_{,y}\,\mathrm{d}y \\ - \int_C [M_{xy}\delta w]_{,x}\,\mathrm{d}x + \int_c \delta F u_{,yy}\,\mathrm{d}y - \int_c \delta F v_{,xx}\,\mathrm{d}x \\ + \int_c \delta F_{,x}v_{,y}\,\mathrm{d}y - \int_c \delta F_{,y}u_{,x}\,\mathrm{d}x + \frac{1+\nu}{Eh}\int_C [\delta F F_{,xy}]_{,y}\,\mathrm{d}y \\ - \frac{1+\nu}{Eh}\int_C [\delta F F_{,xy}]_{,x}\,\mathrm{d}x = 0 \qquad (2)\end{aligned}$$

Using Eq. (2), the Euler-Lagrange equations governing the problem are obtained and read

$$\begin{aligned} D\nabla^4 w + (1/R)F_{,xx} + \rho h w_{,tt} + Q_t(w) \\ + Q_x(w) + P(w) = 0 \qquad (3)\end{aligned}$$

and

$$\nabla^4 F - (Eh/R)w_{,xx} = 0 \qquad (4)$$

and the boundary conditions are obtained as follows.

1) On x = const: w is prescribed or $M_{x,x} + 2M_{xy,y} = 0$, $w_{,x}$ is prescribed or $M_x = 0$, F is prescribed or $u_{,yy} = 0$, and $F_{,x}$ is prescribed or $v_{,y} = 0$.

2) On y = const: w is prescribed or $M_{y,y} + 2M_{xy,x} = 0$, $w_{,y}$ is prescribed or $M_y = 0$, F is prescribed or $v_{,xx} = 0$, and $F_{,y}$ is prescribed or $u_{,x} = 0$.

3) At a corner (discontinuity in C) $M_{xy} = 0$ (equivalent to $w_{,xy} = 0$) if w is not prescribed and $F_{,xy} = 0$, if F is not prescribed.

The first conditions are the forced or geometrical conditions, and the second ones are the free or natural conditions. When using a variational formulation for a boundary value problem, the admissible functions should satisfy only the forced boundary conditions. Therefore, using the listed conditions we can write the classical boundary conditions on an edge $\mu =$ const, where μ stands for x or y, and η is taken as the normal direction to μ, as

$$\text{clamped edge } w = w_{,\mu} = 0, \text{ and at a corner } F_{,\mu\eta} = 0 \quad (5a)$$

$$\text{free edge } F = F_{,\mu} = 0, \text{ and at a corner } M_{,\mu\eta} = 0 \text{ (i.e., } w_{,\mu\eta} = 0) \quad (5b)$$

$$\text{simply supported edge } w = 0, \text{ and at a corner } F_{,\mu\eta} = 0 \quad (5c)$$

$$\text{freely supported edge } w = F = 0 \quad (5d)$$

These boundary conditions deserve some comments. In terms of the in-plane displacements u and v and the transverse displacement w on a clamped edge, $\mu =$ const, we have $u = v = w = 0$ and $w_{,\mu} = 0$. On a free edge we have $u \neq 0$, $v \neq 0$, $w \neq 0$, and $w_{,\mu} \neq 0$. Freely supported end condition stands for an edge which is free to move in the μ direction and, simply restrained in the v and the transverse directions, i.e., $u \neq 0$, $v = 0$, $w = 0$, and $w_{,\mu} \neq 0$. Simply supported edge condition is for $u = 0$, $v = 0$, $w = 0$, and $w_{,\mu} \neq 0$. Traditionally, the freely supported edge conditions have often been used in theoretical shell analysis due to the ease of employing interpolation functions that satisfy the differential equations governing the problem and the boundary conditions. Practical end conditions are free, simply supported, or clamped conditions. In the past great difficulties have been experienced in reproducing conditions in accordance with the theoretical assumptions made for freely supported boundary conditions, this was done, in spite of not representing real boundary conditions, in order to compare the experimental results with the theoretical formulations. The present formulation permits the application of all of the actual boundary conditions in a simple and direct way. Further, as will be shown in the results section, for the shell problem considered here, and depending on the curvature parameter of the shell, the simply supported end conditions have a different behavior as compared to the freely supported end conditions. Now, a finite element solution for the problem at hand can be performed, using rectangular elements preserving C^1 continuity, based on the functional given in Eq. (1). Thus, we can write

$$\zeta(x, y) = \sum_{i=1}^{2}\sum_{j=1}^{2}\left[H_{0i}(x)H_{0j}(y)\zeta_{ij} + H_{1i}(x)H_{0j}(y)\zeta_{,x_{ij}} + H_{0i}(x)H_{1j}(y)\zeta_{,y_{ij}} + H_{1i}(x)H_{1j}(y)\zeta_{,xy_{ij}}\right] \quad (6)$$

where ζ stands for w or F and H_{mn} are first-order Hermitian polynomials. Using the standard finite element technique, we obtain for each element a set of two equations cast in the form

$$[k_{ww}]\{w\} + [k_{wF}]\{F\} + [m]\{\ddot{w}\} + C_1[n_1]\{w\} + C_2[n_2]\{\dot{w}\} = \{0\} \quad (7)$$

$$[k_{Fw}]\{w\} + [k_{FF}]\{F\} = \{0\} \quad (8)$$

The element stiffness matrix $[k_{ww}]$, the compatibility matrix $[k_{FF}]$, the coupling matrices $[k_{wF}]$, and its transposed $[k_{Fw}]$, and the mass matrix $[m]$ can be calculated as given in Refs. 35 and 39. The incremental stiffness and damping matrices $[n_1]$ and $[n_2]$, due to the presence of the follower forces, will be obtained for each case in consideration with a formulation similar to that of Ref. 35. Using the finite element standard assembly technique and applying the appropriate boundary conditions, the matrix equation for the whole structure reads

$$[K_{ww}]\{w\} + [K_{wF}]\{F\} + [M]\{\ddot{w}\} + C_1[N_1]\{w\} + C_2[N_2]\{\dot{w}\} = \{0\} \quad (9)$$

$$[K_{Fw}]\{w\} + [K_{FF}]\{F\} = \{0\} \quad (10)$$

Now, the degrees of freedom $\{F\}$ can be eliminated using the compatibility Eq. (10) and the solution of the problem is reduced to

$$[K_{\text{eq}}]\{w\} + C_1[N_1]\{w\} + C_2[N_2]\{\dot{w}\} + [M]\{\ddot{w}\} = \{0\} \quad (11)$$

where

$$[K_{\text{eq}}] = [K_{ww}] - [K_{wF}][K_{FF}]^{-1}[K_{Fw}] \quad (12)$$

An examination of Eq. (11) reveals that the computational effort required for the solution of the stability problem is equivalent to that of a flat plate problem when the present formulation is used. Further, the in-plane boundary conditions are applied for F, $F_{,x}$, $F_{,y}$, and $F_{,xy}$ and are all nodal degrees of freedom. It is to be observed that the boundary conditions on F and its partial derivatives are performed on Eq. (10) before the application of the static condensation procedure.

III. Applications

We consider in this section a cantilever circular cylindrical panel subjected to a distributed tangential follower force $q(x)$ and a tangential end follower force $q(L)$ as shown in Fig. 1. In this example, the damping contribution is not considered in the analysis, therefore, $Q_t(w)$ is taken equal to zero in Eq. (1). The variation in the work done by the follower force $q(x)$ through a variation in the generalized displacements reads

$$\delta W = -\int_A (L - x)q(x)\frac{\partial w}{\partial x}\delta\frac{\partial w}{\partial x}\,dA + \int_A q(x)\frac{\partial w}{\partial x}\delta w\,dA \quad (13)$$

and the variation in the work done by the follower end tangential force through a variation in the generalized displacements reads

$$\delta W = -\int_A q(L)\frac{\partial w}{\partial x}\delta\frac{\partial w}{\partial x}\,dA + \int_C q(L)\frac{\partial w}{\partial x}\delta w\,ds \quad (14)$$

The nonconservative follower forces $Q_x(w)$ and $P(w)$ of Eq. (1) are directly correlated to the follower loads $q(x)$ and $q(L)$ in Eqs. (13) and (14) by equating the corresponding terms. It is instructive to observe that the follower forces considered here produce loadings that are in part conservative and in part nonconservative. An examination of Eq. (13) reveals that the first integral will produce a conservative loading, and its effect on the system stability can produce only a static mode of instability (i.e., divergence). In fact, this portion of the loading will generate a self-adjoint problem, and its effect is exactly the same as the formulation of the problem under the effect of a state of initial stress. In the finite element formulation, this type of formulation will lead to the geometric stiffness matrix extensively used in linear static buckling analysis. The second integral will produce a nonconservative loading, and its effect on the structure can produce a dynamic mode of instability (i.e., flutter). This portion of loading will generate a nonself-adjoint problem, and its effect is exactly the same as the response of the shell in the presence of an external supersonic flow. The problem of supersonic aeroelasticity of plates and shells has been extensively studied in the literature, see, for instance, the review paper on the subject, Ref. 40. Finally, it is to be noted that, in the finite element method formulation, the first part of the load will produce a symmetric matrix whereas the second part will produce an asymmetric matrix. For the case of a tangential end follower load, the same conclusions are made, and we observe that the first integral in Eq. (14) represents the conservative loading part whereas the second integral represents the nonconservative part of the loading.

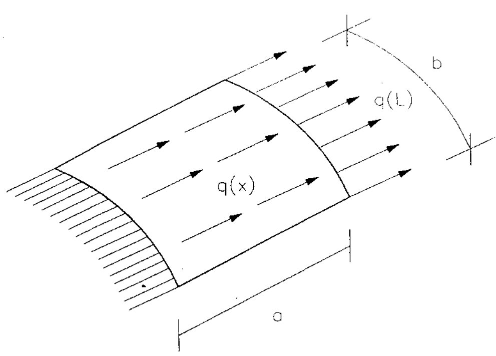

Fig. 1 Cantilever cylindrically curved panel subjected to distributed and end follower forces.

IV. Numerical Results

Several numerical calculations have been performed to show the applicability of the method and its performance. The first set of calculations were done for a cantilever flat plate subjected to a tangential distributed follower force. The plate was analyzed for three boundary conditions for the lateral edges. In the first case the lateral edges were considered free, in the second case the lateral edges were taken as simply supported, and in the third case the lateral edges were considered clamped. The calculations were performed for different values of the plate aspect ratio a/b. The results of the calculations in terms of the nondimensional critical parameter $p_{cr} = q_{cr}(x)a^3/D$, for the different conditions analyzed, are given in Table 1. From these results it can be concluded that the flat plate behaves in a regular way; in all of the cases analyzed the first instability is caused by the coalescence of the first two modes. Further, as was expected, the cantilever flat plate with lateral edges clamped is more stiff than for the other lateral edge conditions and, therefore, presents a critical instability parameter higher than the simply supported and the free lateral edge conditions.

The next series of calculations were done for the cantilever cylindrically curved panel shown in Fig. 1. The calculations were performed for three aspect ratio conditions, namely, $a/b = 1$, 1.5, and 2. The results of these computations are given in Figs. 2, 3, and 4, respectively. In each case of aspect ratio three sets of numerical calculations have been performed for the cylindrically curved panels. In the first set of calculations, the lateral edges of the cantilever shell were considered free. The second set of calculations were performed considering these edges to be freely supported. In the third set, the lateral edges were considered simply supported. The results are plotted for the nondimensional critical load $p_{cr} = q_{cr}(x)a^3/D$ vs the curvature parameter z for the different cases analyzed. From the results of Figs. 2–4, it can be concluded that for a shell with a very small curvature, the behavior of the shell is similar to that of a flat plate. In this region the curvature effect is stabilizing in the sense that the critical stability parameter increases with the increase of the curvature. With further increase of the curvature, the panel passes through a transition region characterized from a flat plate behavior to a deep shell behavior. This region is characterized by the dips and cups observed in the curves of the critical stability parameter vs the curvature parameter and is explained by coalescence of successive higher modes to produce the first critical instability condition. After this transition region, with further increase in the curvature, the panel behaves as a deep shell, and the critical instability modes are those with an elevated number of waves in the radial direction and the first longitudinal modes. In this part the shallow shell theory is no longer adequate, and deep shell theory must be used. The present shallow shell theory is, therefore, limited to the flat plate region and the transition part behavior of the curved panels. Concerning the aspect ratio effect it can be concluded that in the plate region the effect of the aspect ratio is stabilizing. Further, in this region, i.e., for a small curvature parameter, the shell with freely supported end conditions is more stable than a shell with simply supported and with free lateral edges. It is again emphasized that the freely supported conditions are only theoretical conditions that do not represent real structure. In the transition region, i.e., with moderate curvature effect, the stability parameter is characterized by the successive coalescence of higher modes. In this region no definite trend can be made on the effect of curvature and/or aspect ratio on the critical stability parameter. Again, this transition region is physically explained by the spectrum of the free vibration modes of the structure.

As a final set of calculations the cantilever cylindrically curved panel, subjected to distributed follower force and clamped on the

Table 1 Critical load $P_{cr} = q_{cr}a^3/D$ for cantilever flat plates having lateral edges clamped, simply supported, and free

a/b	P_{cr} Clamped	P_{cr} Simply supported	P_{cr} Free
0.50	51.10	48.03	29.79
1.00	89.42	79.72	35.00
1.25	113.44	102.20	35.57
1.50	146.66	127.75	35.82
1.75	180.38	156.37	35.97
2.00	217.18	184.47	36.23
2.25	257.03	220.24	36.03
2.50	299.96	256.01	36.03
2.75	346.97	294.85	36.02
3.00	397.05	335.73	36.02

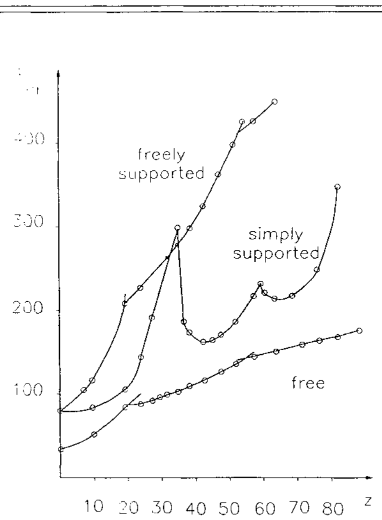

Fig. 2 Critical load $P_{cr} = qa^3/D$ vs curvature parameter $z = (a^2/Rh)(1 - \nu^2)$ for a cantilever shell with lateral edges free, freely supported, and simply supported for an aspect ratio $a/b = 1$.

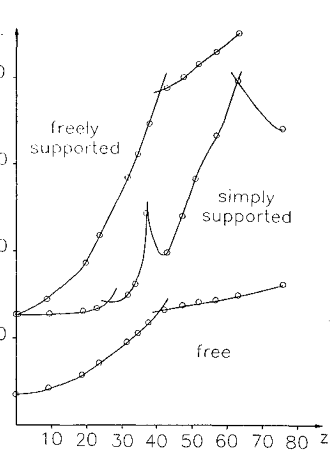

Fig. 3 Critical load $P_{cr} = qa^3/D$ vs curvature parameter, $z = (a^2/Rh)(1 - \nu^2)$ for a cantilever shell with lateral edges free, freely supported, and simply supported for an aspect ratio $a/b = 1.5$.

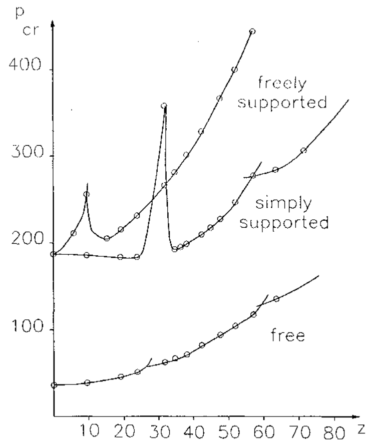

Fig. 4 Critical load $P_{cr} = qa^3/D$ vs curvature parameter $z = (a^2/Rh)(1 - \nu^2)$ for a cantilever shell with lateral edges free, freely supported, and simply supported for an aspect ratio $a/b = 2$.

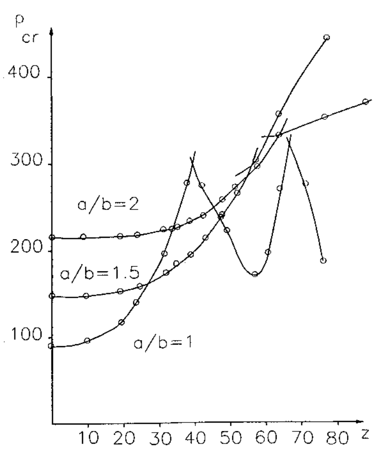

Fig. 5 Critical load $P_{cr} = qa^3/D$ vs curvature parameter $z = (a^2/Rh)(1 - \nu^2)$ for a cantilever shell with lateral clamped for an aspect ratio a/b = 1, 1.5, and 2.

lateral edges, is analyzed. The calculations were performed for three aspect ratio conditions, namely, $a/b = 1, 1.5$, and 2, and for different curvature parameter values. The results of the calculations are shown in Fig. 5. From these results, it can be concluded that the clamped lateral edge condition is more stable than the other end conditions, and the shell conserves the same behavior already explained concerning the aspect ratio and the curvature effects.

V. Discussion

Several general conclusions are made in this section concerning the stability of cylindrically curved panels subjected to follower forces.

1) The response of the shell to a follower force presents the same behavior as that for supersonic flutter of the shell in the presence of a state of initial stress. Traditionally, in the aeroelastic analysis of plates and shells, the effect of a prestress load is considered as a parameter of the problem. In other words, for a given value of the initial stress, the dynamic pressure is varied until obtaining the critical value for this predefined initial stress load level. In the problem of follower forces treated here, the prestress load bears a ratio of the nonconservative part of the load, i.e., for each change in the follower force intensity, the geometric stiffness matrix is correspondingly factored by a constant proportional to the value of the follower force. In the aeroelastic problem, this will be equivalent if the prestress load is proportional to the dynamic pressure parameter. Physically, this may represent a load due to aerodynamic heating, for instance.

2) For the shell problem analyzed here, the first instability to occur is of a dynamic nature, i.e., flutter due to coalescence of modes. However, this cannot be generalized, since whether a static (divergence) or dynamic (flutter) instability will occur first is a function of the shell geometry, its boundary conditions, and the type of the follower force applied.

3) A complication in shell stability analysis is that the first modes are not necessarily the critical modes for stability. This is reflected by the discontinuities in the curves of Figs. 2–5 and are explained by successive coalescence of higher modes of the shell to produce the critical loads as the curvature parameter increases.

4) As a consequence of item 3, it is concluded that, if a numerical method is used for the analysis, the representation and the mode extraction technique used must have the same precision for the treatment of higher modes, which may cause the instability, as for lower modes. The present finite element formulation does preserve this property.[35–38]

5) For clarity of the exposition, no damping has been incorporated in the analysis. If a constant viscous type damping is used, it can be shown that its effect is always stabilizing in the sense of increasing the critical load. For other types of damping, e.g., structural damping, which are strain and, therefore, stress dependent, their effect may be stabilizing or destabilizing.[38,40]

VI. Conclusions

A dynamic stability analysis of cylindrically curved shallow shells subjected to follower forces has been presented. The shell analysis is based on Reissner's two-field variable variational principle. It is shown that the boundary conditions for the Airy stress function are as simple and direct to apply as those for the transverse displacement. The element used in the analysis is characterized by its high precision and direct application of the boundary conditions. The similarity between the response of the shell to nonconservative follower forces, and its response in the presence of a supersonic flow has been demonstrated.

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